RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022
FIRST YEAR [BATCH 2021-24]

Date : 12/03/2022 — **MATHEMATICS (General)**
Time : 11am-1pm — **Paper** : MAGT 1 — Full Marks : 50

Group:A

Answer **any three** questions of the following: [3 × 6=18]

1. The equation $3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$ is reduced to $4x^2 + 2y^2 = 1$, when referred to rectangular axes through the point $(2, 3)$. Find the inclination of the latter axes to the former. [6]

2. Show that the equation $x^2 - 2y^2 + xy - 7x + 10y - 8 = 0$ represents an intersecting pair of real straight lines, and find their point of intersection. [3+3]

3. Prove that if the straight line $lx + my + n = 0$ touches the parabola $y^2 - 4px + 4pq = 0$, then $l^2q + ln - m^2p = 0$. [6]

4. If PSP' and QSQ' are two perpendicular focal chords of a conic, then prove that $\frac{1}{PS \cdot SP'} + \frac{1}{QS \cdot SQ'}$ is a constant. [6]

5. Find the centre and radius of the circle $x^2 + y^2 + z^2 - 2y + 2z - 2 = 0$, $x - y + z = 1$. [3+3]

Group: B

Answer **any four** questions of the following: [4 × 8=32]

6. (a) State De Moivre's theorem on complex numbers. [2]
   (b) Find the sum of 99 th. power of the roots of the equation $x^7 - 1 = 0$. [3]
   (c) Prove that $Cos^{-1}(2) = 2n\pi \pm i log(2 + \sqrt{3}), n \in \mathbb{N}$. [3]

7. (a) If α is a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0 (d \neq 0)$. Prove that $\alpha = -\frac{8d}{3c}$. [4]
   (b) Solve the equation $16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0$ whose roots are in arithmatic progression. [4]

8. (a) Prove that $Log(i) = \frac{4n+1}{4m+1}$ where $m, n \in \mathbb{N}$. [4]
   (b) Solve the equation $(1 - x)^n = (1 + x)^n$. [4]

9. (a) Define a group. [3]
   (b) Prove that every subgroup of a cyclic group is cyclic. [5]

10. (a) Show that the subset $S = \{(1,3,-4,2),(2,2,-4,0),(1,-3,2,-4),(-1,0,1,0)\}$ of $\mathbb{R}^4$ is linearly dependent. [5]

(b) Let u, v and w be distinct vectors of a vector space V_F. Show that if $\{u, v, w\}$ is a basis for V_F, then $\{u+v+w, v+w, w\}$ is also a basis for V_F. [3]

11. (a) State Caley-Hamilton theorem. [2]

(b) Find all the eigen-values and eigen-vectors of the following matrix.

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{pmatrix}$$

[6]

$- - - - \times - - - -$